# On Axiomatization of Plurality Decisions with Veto 

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#### Abstract

The article presents an analysis of the axioms associated with the plurality method of aggregation of individual preferences, both when it is necessary to select one of many alternatives and when it is necessary to approve a single alternative. Also, we investigate the impact of the introduction of a new attribute, being the right of veto (absolute and relative), on the axioms given. In the conclusion, the emphasis is that the commonly used method of aggregation, i.e. the plurality method is not, in this sense, the best method, .


Keywords: aggregation of preferences, axioms, power indices. JEL: C71, D71, D72.

## Introduction

Making group decisions ${ }^{1}$ occurs in multiple decision-making situations. It occurs when decisions are made by people (e.g. voters, parliamentarians or supervisory board members), as well as when they are made by soft-ware-embedded machines (e.g. multiple agents systems, image analysis techniques or decision-support systems). The decision-making rule that is applied most frequently in those situations is the majority rule (simple

[^0]History: Otrzymano 2016-03-21, poprawiono 2016-03-29, zaakceptowano 2016-06-01
or qualified). This rule implies that if an alternative should be chosen from a set of alternatives, then the alternative for which the majority of deci-sion-makers opted is chosen. Every specific situation requires that the notion of "majority" be defined and a method for settlement provided when a majority cannot be reached, yet, a decision must be taken. An additional and frequently deployed decision scheme is to endow some of de-cision-makers with the so called right of veto. This right means that one or more decision-makers can defer the choice of a particular alternative absolutely (that is forever) or relatively, where the decision to use veto can be overruled. Obviously, adding veto
changes the importance of some or all of decision-makers ${ }^{2}$. In the paper, we intend to analyze a set of axioms describing such decision-making situations and outline how adding the right to veto affects the selected axioms.
The structure of the paper is as follows: the first chapter presents the idea behind group decision with veto. Depending on the amount of alternatives from which the choice is to be made, subsequent chapters analyze the set of postulates (axioms) associated with the plurality aggregation. The second chapter includes a discussion on axioms associated with the plurality choice of one out of more than two alternatives. The components of simple plurality voting games theory (voting theory) are introduced in the third chapter. Moreover, a minimum set of plurality decision axioms together with the relevant theorems on non-existence or existence of aggregation functions are also discussed. The paper ends with a conclusion and proposals for further research.

## Initial Presumptions of Group Decisions with Veto

For the analysis of group aggregation methods, the so called individual profiles provide a starting point. They state that every decision-maker is capable of ordering (partially or completely) a set of alternatives using their own function of preferences.
The plurality method of aggregation is to be understood as a way of aggregating individual profiles, where every decision-maker indicates their best alternative, and the alternative that is going to be chosen is the one which gets the highest amount of votes ${ }^{3}$.

[^1]Plurality aggregation with veto occurs when at least one decision-maker has the right to veto the choice made. In the paper, we examine decisions where the decision-makers have to choose one alternative from a set of alternatives $W=\left\{w 1, w 2, \ldots, w_{r}\right\}$, $r=1,2, \ldots, n$. Analyzing group decisions, we will investigate, due to the amount of alternatives, two classes of decisions, $\mathrm{K}_{1}$ and $\mathrm{K}_{2}$. The first class $\mathrm{K}_{1}$ is made up of the decisions in which an $N$ number decision-makers participate ( $|N|=n \geq 2$ ), having to choose one alternative from the set of alternatives $W,|W|=m \geq 2$. The second class, $\mathrm{K}_{2}$, is made up of the decisions in which an $N$ number decision-makers take part $(|N|=n \geq 2)$ who have to accept a single alternative $(|W|=1)$. The investigation of group aggregation methods should be carried out in the following ways:

1. By characterizing individual aggregation methods, describing the conditions in which they provide a clear-cut result, etc.
2. By specifying a set of criteria for which no aggregation method exists capable of providing clearcut results and simultaneously fulfilling all those criteria, and
3. By examining aggregation methods in terms of their satisfying or failing to satisfy a particular criterion (axiom).
With respect to the decision made in class $\mathrm{K}_{1}$, we assume that every
requirement with respect to the number of votes which needs to be fulfilled so that a particular choice could be accepted. Amongst the classical plurality methods we can distinguish, e.g. plurality with run off method, where $50 \%$ of votes is reached by (if it is necessary) eliminating sequentially alternatives which receive the lowest amount of support; Hare method, where individual profiles do not get changed while applying plurality with run off method, or Coombs' method, which is in fact the reverse of Hare method.
decision-maker is capable of ordering a set of alternatives (in the best case by individual preference relation: either weak or strong). In the worst case the ordering unfolds by comparing pairs with the admissible partial order and equivalent alternatives ${ }^{4}$.
With respect to class $\mathrm{K}_{2}$, we assume that every decision-maker can make a decision whether or not to accept a particular alternative ${ }^{5}$. All de-cision-makers present their views on a particular alternative (e.g. by voting) and if it receives the required amount of support (votes), then it is accepted by the group. For this purpose we introduce the following notation derived from simple games.
Let $N$ represent a finite set of deci-sion-makers, $\boldsymbol{q}$ the required number of votes (quota) to make a decision and $w_{j}$ denotes the weight of deci-sion-maker $j, j \in N$. Let us consider a special class of cooperative games called weighted majority games. Weighted majority game G is defined by $\boldsymbol{q}$ and a sequence of non-negative weights $\omega_{i}, i \in N$. We can think of $\omega_{i}$ as of number of votes of an $i$-th decision-maker, and of $\boldsymbol{q}$ as number of votes necessary for an alternative to be collectively accepted, that is, necessary for the group of decision-makers to become a winning group (coalition). In the interest of simplicity, we assume that both $\boldsymbol{q}$ and $\omega_{i}$ are positive integers. Any subset of decision-makers is named a coalition.
The decision acceptance is thus equivalent to the formation of a winning coalition of decision-makers. Simple game $(N, v)$, where $N$ is a set of decision-makers

[^2](players) and $v$ is a characteristic function of the game ${ }^{6}$, is only then a proper game when the following condition is fulfilled: for all coalitions $T \sqsubset N$, if $v(T)=1$, then $v(N \backslash T)=0$. This means that decisions are made by the majority of at least $\boldsymbol{q}$ votes $((0,5<q \leq 1)$. Let us consider only simple games (SG) where a deci-sion-maker (player) may vote yes-no, or yes-no-abstain. The last one is an analogue of a weak preference where there is a greater number of alternatives. Alternatively, we can describe the decision-making body as a triplet $(N, q, v)=\left(N, q, \omega_{1}, \omega_{2}, \ldots, \omega_{n}\right)$.
If a particular member of the deci-sion-making body can transform the winning coalition into the non-winning one by utilizing veto, then this type of veto is called veto of the first degree. If a particular member of the decision-making body can transform only some of the winning coalitions, that is, not all of them, into the non-winning ones, even without being their member, then this type of veto is called veto of the second degree (Mercik, 2011).
Let us note that the behaviour of the decision-makers in the winning coalition can provide a basis for the evaluation of their impact on the final outcome measured by the so called power index. From a formal point of view, the representation $\varphi: S G \rightarrow R^{n}$ is called power index. For each $i \in N$ and $v \in S G$, the $i$-th coordinate $\varphi(v) \epsilon R^{n}, \varphi(v)(i)$ is interpreted as the voting power of player $i$ in game $v$. In general, power indexes are a priori in their nature. Potential situations are analysed in which individual decision-makers (players) change their mind (e.g. from voting yes to voting no) and how this change affects the position of a given

[^3]player. If a particular member of a winning coalition transforms the winning coalition into a non-winning one by changing the vote, then he/ she is in the position called swing position (which leads to Penrose-Banzhaf power index (1965), with the coalition being "sensitive" to the behaviour of a particular decision-maker, which, in turn, leads to Johnston power index (1978). If, however, the decision-maker changes his/her mind and joins a particular coalition and thus transforms it into a winning coalition, then he/she is a pivotal player, which leads to Shap-ley-Shubik power index (1954) ${ }^{7}$.
Penros-Banzhaf power index (1965) for a simple game is a value equal to: $\varphi: S G \rightarrow R^{n}$, $v \rightarrow(\varphi 1(\mathrm{v}), \varphi 2(\mathrm{v}), \ldots, \varphi \mathrm{n}(\mathrm{v}))$, wherefor every $i \varepsilon N ; \operatorname{card}\{N\}=n ; \operatorname{card}\{S\}=s$
$\varphi_{i}^{B}(v)=\frac{1}{2^{n-1}} \sum_{S \subseteq N\{i\}}[v(S \cup\{i\})-v(S)]$
Moreover, Shapley-Shubik power index (1954) represents the following value: $\varphi: S G \rightarrow R^{n}$, $v \rightarrow(\varphi 1(\mathrm{v}), \varphi 2(\mathrm{v}), \ldots, \varphi \mathrm{n}(\mathrm{v}))$, where for every $i \varepsilon N$, card $\{N\}=n$; $\operatorname{card}\{S\}=s$
$\varphi_{i}^{S S}(v)=\sum_{S \subseteq N\{i\}} \frac{s!(n-s-1)!}{n!}$
The above values are obtained directly from the games defined with the use of the characteristic function, where marginal values of the power increase are calculated for every winning coalition.
We will demonstrate that deploying the plurality aggregation method and including veto results in the lack of an unequivocal set of axioms pertaining

[^4]to all sets of postulates concerning the behaviour of decision-makers. Possible assertions that there exists or does not exist an optimal group aggregation method (including the plurality method) always depend on the selected set of axioms. This also applies to potential measurement of the power of the decision-maker in the plurality method.

## Axiomatization of Decisions in Class $\mathrm{K}_{1}$

Let us recall that the first class, $\mathrm{K}_{1}$, is made up of decisions in which an $N$ number of decision-makers participate ( $|N|=n \geq 2$ ), having to choose one alternative from the set of alternatives $W$, $|W|=m \geq 2$.
The fundamental assumption of the theory of collective decisions is the rationality of the decision. At the level of individual decision-making, this implies, in crude terms, that if a particular decision-maker considers the choice of a given alternative to be the best (assuming that there are more alternatives than just one), then he/she will chose this alternative.
If, however, the decision-maker cannot indicate a single alternative, then it means that there are at least two alternatives which he/she considers to be the best, according to his/her own criteria. If the choice of a single alternative is still necessary, then we agree on some way of settling such "tie" situations, e.g. by drawing lots or resorting to some other criterion (e.g. alphabetical order of the names of the alternatives). However, there should be no doubt that such decisions are rational in the sense that a better alternative (or at least not worse than the remaining ones) gets always chosen.
We thus assume, from a formal viewpoint, that, between a pair of alternatives a and b coming from a given set of alternatives $W$, one of three
possible preference relations occurs $(\succ): a \prec b, b \prec a$ or $a \sim b$ (strong alternative) or $a \preccurlyeq b, b \preccurlyeq a$ or $a \sim b$ (weak alternative). We usually assume with respect to this kind of preference that it is reflexive, transitive and complete. ${ }^{8}$ The rationality of group decisions, i.e. decisions made by more than one de-cision-maker is no longer so obvious, since what seems rational at a level of individual decision-making need not be so at the level of a group. The aggregation of individual decisions can lead to the choice of an alternative which is not the best in terms of the criteria used. Although Condorcet (1785) already introduced the two main criteria ${ }^{9}$ which the group decision-making should fulfil, what soon followed is that the majority of decision-making methods do not satisfy those criteria, resulting in what is known in literature as a voting paradox.
Let us then take a closer look at the axioms characterizing the rationality of the aggregation of group decisions. It is assumed (just like for the preferences describing individual de-cision-making) that the group preference relation is rational if it satisfies the requirements of the reflexive, complete and transitive relation. Unfortunately, the Condorcet's paradox ${ }^{10}$
$\overline{8}$ That this assumption is rather idealistic has been demonstrated in the research on human behaviour, where the majority of created orders do not retain transitivity (e.g. Aumann, 1985 or Gilboa et al., 2012).
${ }^{9}$ Two main criteria of Condorcet are: (1) if a particular alternative wins in a pairwise comparison with the remaining alternatives, then it is the winning alternative, according to a particular method of group voting, and (2) if a particular alternative loses in a pairwise comparison against other alternatives, then it cannot be the winning alternative of a given group voting method.
${ }^{10}$ An example of the paradox which leads to a tie is the situation where three decision-makers $\mathrm{A}, \mathrm{B}$, and C have the
shows that the group relation is not always transitive even if individual preferences are transitive.
It is expected (e.g. Mercik, 1998, Lissowski, 2008) that the employed method of the individual preferences aggregation should be immune to manipulations (individual ${ }^{11}$ and agenda manipulations ${ }^{12}$ ), be effective computationally (i.e. it should lead to choosing one alternative) and it should satisfy the so called Arrow's axioms, that is: (1) it should be defined as a set of all possible individual voter preferences (postulate of unrestricted domain), (2) it should fulfil the requirement of Pareto optimality (at least in its weak version) ${ }^{13}$, should be independent of irrelevant alternatives ${ }^{14}$, and (4) there should be no decision-maker who is a dictator.
Arrow (1951, the so called impossibility theorem) showed that if there are at least two decision-makers and at least three alternatives to be voted on, then there is no method of group aggregation which would satisfy the aforementioned criteria. Similar to that, Gärdenfors (1976) shows that there exists no aggregation technique that could satisfy both Condorcet's
following preferences regarding three alternatives, and ,respectively.
${ }^{11}$ Group aggregation method is individually manipulable if the decision-maker changes his/her preferences purposefully and as a result achieves the desired outcome of the aggregation.
12 The aggregation method is agenda manipulated if, e.g. the inclusion of a new alternative or the change of a given voting order changes the result of the aggregation. ${ }^{13}$ If the decision-makers are in full agreement which alternative is the best and which is the worst, then the particular aggregation method should only refer to the remaining alternative (if we want a complete ordering (also the weak one) of all alternatives).
14 Irrelevant alternatives are alternatives outside the alternatives whose ordering we are considering.
criteria for weak preferences, and Gibbard (1973) and Satterthwait (1975) demonstrated that every rational, in Arrow's sense, aggregation method is individually manipulable.
As the subject of discussion in the paper is the aggregation method called the plurality method, we must at once see that this is not the method which could satisfying Arrow's postulates. May (1952) named necessary and sufficient conditions (unanimity ${ }^{15}$, duality ${ }^{16}$, and strong monotonicity ${ }^{17}$ ) whose fulfilment allows one to chose one alternative in unanimous way by applying the plurality method.
In Mercik's work (1990) we may see that the plurality method of aggregation satisfies the following axioms:

- Condorcet's axiom I on choosing the wining alternative in a pairwise comparison,
- monotonicity axiom,
- consensus axiom ${ }^{18}$,
- axiom of simplicity and easy application, and
- Pareto axiom

This represents 6 out of 9 of the axioms under discussion. "The best" of the aggregation methods (if all the axioms are treated equally) satisfies 7 axioms (approval method ${ }^{19}$ ). As

[^5]we can see, none of the aggregation methods satisfy each axiom.
In this case introducing veto is equivalent to establishing a dictator whose decisions are irrevocable (veto of the first type) or conditional (veto of the second type). In none of the sets of axioms proposed for specifying the theorems about the existence or non-existence of group aggregation method the dictator is admissible (veto of the first type). The conditional veto (veto of the second type), in the plurality method (where only the alternatives ranked first in the individual orderings by the decision-makers are of importance), implies raising the limit on the amount of necessary first rankings for a particular alternative and almost inevitably means that a plurality-at-large method will have to be applied (Ramsey, Mercik, 2015). This, however, does not affect the fulfilment of specific axioms by this method.

## Axiomatization of Decisions in Class $\mathrm{K}_{2}$

Let us recall, the second class, $\mathrm{K}_{2}$, is made up of the decisions in which an $N$ number of decision-makers take part ( $|N|=n \geq 2$ ), having to accept a single alternative.
For $\mathrm{n}=2$ the application of the plurality method means that it is necessary for alternatives to be accepted by both decision-makers simultaneously. The right of veto does not have any impact on the outcome of the aggregation. Significant changes occur when we raise the number of decision-makers ( $n>2$ ), which is what we will discuss presently.
We are focusing on the question of accepting an alternative based on group
disqualifies the very method. However, the incidence of paradoxes in this method of aggregation is so low (Nurmi, Uu-si-Heikkilä, 1985) that it can practically be disregarded.
decision. In this case, it is of significance to try and specify the possibilities to exert influence on such a decision, so that it would be in accordance with a particular decision-maker's preference. Naturally, it is not possible to examine how a specific deci-sion-maker will behave (apart from a post factum analysis), yet, we could employ here an a priori analysis, assuming that every decision-maker knows whether they are for or against a particular alternative.
Let us investigate what axioms are linked to an a priori power index, regardless of its form. As we have already mentioned, a priori power of a given decision-maker can be measured using the so called power indices. The following representation we call power index: $\varphi: S G \rightarrow R^{n}$. For every $i \in N$ and $v \in S G$, the $i$-th coordinate $\varphi(v) \in R^{n}, \varphi(v)(i)$, is interpreted as the decision-maker's power i in the game with a characteristic function $v$. One may expect that the decision-maker endowed with the right to veto $i_{\text {veto }}$ should have a priori power at least not smaller than without this right. This leads to the first axiom called a value-added axiom: $\varphi(v)\left(i_{\text {veto }}\right) \geq \varphi(v)(i)$. For the veto of the first type we get: $\varphi(v)\left(i_{\text {veto }}\right)>\varphi(v)(i) \quad$ 20. Of course, for certain situations with the veto of the second kind we get: $\varphi(v)\left(i_{v e t o}\right)=\varphi(v)(i)$.
The second axiom is the so called gain-loss axiom: if $\varphi(v)(i)>\varphi(w)(i)$ for $v, w \in S G$ and $i \in N$ there exists $j \in N$ such that $\varphi(v)(j)<\varphi(w)(j)$. Let us notice that if $\varphi(v)\left(i_{\text {veto }}\right)>\varphi(w)(i)$ for $v, w \in S G$ and $i_{\text {veto }} \in N$ there exists $j \in N$ such that $\varphi(v)(j)<\varphi(w)(j)$.
The third axiom refers to the normalization of the power index value:
$\sum_{i \in N} \varphi(v)(i)=1$

[^6]The fourth axiom is called a transfer axiom:
$\varphi(v \vee w)(i)+\varphi(v \wedge w)(i)=$ $=\varphi(v)(i)+\varphi(w)(i)$,
for $v, w \in S G$ and it does not change its properties for games with or without veto. Its equivalent can be expressed (Dubey et al. 1981) as follows: consider two pairs of games $v, v^{\prime}$ and $w, w^{\prime}$ in SG, each game with veto. Let us assume that the transition from $v^{\prime}$ to $v$ and $w^{\prime}$ to $w$ denotes the same winning coalitions, i.e. $\geq v^{\prime}, w \geq w^{\prime} \operatorname{oraz} v-v^{\prime}=w-w^{\prime}$. Hence, the equivalent transfer axiom is as follows:
$\varphi(v)(i)-\varphi\left(v^{\prime}\right)(i)=$
$=\varphi(w)(i)-\varphi\left(w^{\prime}\right)(i)$. This implies that the change in power depends only on the change in the game itself The fifth axiom is called symmetry axiom and denotes that $\varphi(\pi v)(i)=$ $=\varphi(v)(\pi(i))$ for every permutation of players (decision-makers) with or without veto. What is more, changing the way decision-makers are ordered has no effect on the value of their power index.
Once again there is an axiom which is equivalent to the symmetry axiom, and that is the equal treatment axiom (the sixth axiom): if $i, j \in N$ are players in game $v \in S G$ with veto, then:
$S \subset N \backslash\{i, j\} \quad v(S \cup\{i\})=v(S \cup\{j\})$, then $\varphi(v)(i)=\varphi(v)(j)$,
being also valid for $i_{\text {veto }}$ and $j_{\text {veto }} \in N$
All the above axioms are valid for simple majority voting games with veto. However, things look different with the group of further axioms which, although being valid for simple majority voting games without veto, are no longer satisfied when veto has been added to them (Mercik, 2015).
The sixth axiom refers to the so called null player: if $i \in N$ and $i$ is a null player in $v$, i.e. $v(S \cup\{i\})=v(S)$ for every $S \subset N \backslash\{i\}$, then $\varphi(v)(i)=0$. It can be demonstrated that since $\varphi(v)($ iveto $)>0$, than the "null" player axiom can be violated.

The seventh axiom (dummy) seemingly refers to the same as the "null" player axiom: if $v \in S G$ and $i$ is a dummy in game $v, v(S \cup\{i\})=v(S)+v(i)$ for every coalition $S \subset N \backslash\{i\}$, then $\varphi(v)(i)=v(\{i\})$. Here, too, applying veto violates this axiom.
The eighth axiom is concerned with the local monotonicity (or at least the local one): if the weight of the $i$-th player is greater than that of the $j$-th player, then the power index of the $i$-th player cannot be smaller than the power index of the $j$-th player. Neither this axiom is satisfied in the case of simple plurality games with veto.

## Summary

The analysis of decision-making situations involving the aggregation of preferences of individual decision-makers by the plurality method leads to the following conclusions as regards the axioms involved in it:

1. There is no common set of axioms which could be useful in all deci-sion-making situations with plurality aggregation.
2. Individual rationality of deci-sion-makers does not guarantee that there will be group rationality in the decision-making.
3. The plurality method, although simple to use and commonly applied, violates, in many instances, the basic axiom concerning the
acceptance of outcomes obtained on the basis of pair comparisons.
4. Endowing the decision-maker (or decision-makers) with the right of veto leads to further violation of the rules accepted as a rational behaviour of decision-makers.
5. The application of veto of the second type, that is, the one which can be overruled, is equivalent to raising the threshold in the plurality decision-making, however, in principle, it does not change the characteristics of the plurality method itself.
It should be assumed that this situation concerning the satisfaction of the axioms of rational decisions with respect to the plurality method will not change. Nor does it look as if some other new power indices were to emerge to measure the influence a particular decision-maker exerts on the plurality method concerning the acceptance (or the lack of it) of a single alternative. Hence, a rational solution to the aggregation issue of individual preferences should be rejecting the plurality method (including the plurality method with veto) and to explore other and, in this sense, better and more effective methods of aggregation. From the point of view of axioms describing group decisions and power indies, the approval method appears to be a candidate for the aggregation method.

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## O aksjomatyzacji decyzji większościowych z wetem

## Abstrakt

W artykule przedstawiono analizę aksjomatów związanych z większościową metodą agregacji preferencji indywidualnych zarówno wtedy, kiedy konieczny jest wybór jednego z wielu wariantów jak i wtedy, kiedy konieczne jest
zaaprobowanie danego pojedynczego wariantu. Rozpatrzono także wpływ na podawane aksjomaty wprowadzenie nowego atrybutu jakim jest prawo weta (bezwzględnego jak i względnego). W konkluzji podkreślono, że stosowana powszechnie metoda agregacji, tj. metoda większościowa nie jest w tym sensie metodą najlepszą.

Słowa kluczowe: agregacja preferencji, aksjomaty, indeksy siły


[^0]:    ${ }^{1}$ In various fields such decisions are variously named. E.g. in social choice theory one refers to social preference or social decision [Lisowski, 2008].

[^1]:    ${ }^{2}$ The example of real absolute right of veto can be found, among others, in the work of Mercik (2009), while the relative veto in Ramsey, Mercik (2015).
    ${ }^{3}$ In practice the majority rule is often modified, usually by adding an additional

[^2]:    ${ }^{4}$ Interesting findings on ordering equivalent objects with partial orders can be found, e.g. in works of Bury, Wagner (2008).
    ${ }^{5}$ In simple game theory, this corresponds to yes or no voting

[^3]:    ${ }^{6}$ This means that if $\Sigma_{k \in \pi \subseteq N} \omega_{k} \geq q$, then $v(T)=1$

[^4]:    ${ }^{7}$ It is worth noting that the aforementioned power indices (and a variety of other indices) are not equivalent and their application depends on the context of a decision-making situation.

[^5]:    15 If all decision-makers regard a particular alternative as the best (worst), then a given aggregation method also retains it.
    16 If we assume that a given alternative which is individually the best becomes individually the worst (reversal of preferences), then as a result of aggregation this alternative will be the worst with the previous individual preferences.
    17 If two alternatives have been ordered by all decision-makers individually, then the aggregation method retains this order. ${ }^{18}$ This axiom states that if in two separate groups of decision-makers a given alternative is the best in each of the group, then after combining these groups this alternative continues to be the best.
    19 The missing axioms of the approval method are represented by Condorcet's criteria, which, to some extent,

[^6]:    ${ }^{20}$ Let us note that value $\varphi(v)($ iveto $)-\varphi(v)(i)$ reflects well the net value of veto

